You are given a string num, representing a large integer. Return *the* ***largest-valued odd*** *integer (as a string) that is a* ***non-empty substring*** *of* num*, or an empty string* "" *if no odd integer exists*.

A **substring** is a contiguous sequence of characters within a string.

**Example 1:**

Input: num = "52"  
Output: "5"  
Explanation: The only non-empty substrings are "5", "2", and "52". "5" is the only odd number.

**Example 2:**

Input: num = "4206"  
Output: ""  
Explanation: There are no odd numbers in "4206".

**Example 3:**

Input: num = "35427"  
Output: "35427"  
Explanation: "35427" is already an odd number.

**Constraints:**

* 1 <= num.length <= 105
* num only consists of digits and does not contain any leading zeros.